

A New Similarity Measure of q-Rung Orthopair Fuzzy Sets based on the Jaccard Index with its Application to the Orthopairian TOPSIS Method

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ABSTRACT

This paper introduces a novel similarity measure for q-rung orthopair fuzzy sets (q-ROFSs) based on the Jaccard index, aimed at improving decision-making processes under uncertainty. The q-ROFSs are flexible generalizations of intuitionistic fuzzy sets and Pythagorean fuzzy sets, offering improved flexibility in representing uncertainty. Hence, we proposed a novel SM using the Jaccard index for q-ROFSs. The suggested similarity measure considers both the membership and non-membership degrees of q-ROFSs and adheres to key properties such as boundedness, commutativity, and sensitivity. Numerical examples validate the measure, showing it to be both reasonable and effective. The proposed similarity measure is further integrated with the technique for order of preference by similarity to ideal solution (TOPSIS) method, forming orthopairian TOPSIS (OTOPSIS), which enables more accurate ranking of alternatives in uncertain environments. A practical example demonstrates the method's effectiveness, emphasizing its potential for multi-criteria decision-making applications in complex scenarios. The results confirm that the proposed measure is well-suited for decision-making in uncertain, multi-attribute settings.

1. Introduction

Fuzzy sets (FSs) are a concept that was proposed by Zadeh [1]. Since then, FSs have found successful applications in various fields, like image processing [2], management science [3], belief functions [4], etc. Recently, the measurement of heavy metal concentration and health risk index with the help of fuzzy technique for order of preference by similarity to ideal solution (TOPSIS) approach was proposed by Saif-Ud-Din *et al.* [5]. Sanayei *et al.* [6] suggested a group decision-making procedure for supplier selection.

The characterization of FS is based on the belongingness degree and the non-belongingness degree. In FSs, the concept of hesitancy was missing, as it exists in many situations in real life. Therefore, to overcome this issue, Atanassov [7] developed a new generalization of FSs known as

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intuitionistic fuzzy sets (IFSs). The intensity of belongingness, non-belongingness, and the intensity of hesitancy, with the constraint condition, determine how IFSs are to be interpreted. IFSs are widely used in different areas like pattern recognition [8], medical diagnosis [9], etc. Intuitionistic fuzzy TODIM is a multicriteria decision-making tool which is used by Abbas *et al.* [10] to find intuitionistic fuzzy entropy.

In practice, it is observed that there are certain sets which cannot be handled by IFSs due to their constraint condition. Therefore, Yager [11] put forward a new generalization of FSs, namely Pythagorean fuzzy sets (PFSs). Since PFSs are bounded by constraint conditions, some categories of sets cannot be managed by PFSs [12]. To overcome this problem, Yager & Alajlan [13] proposed an extension of PFSs known as q-rung orthopair fuzzy sets (q-ROFSs). q-ROFSs have greater capabilities than IFSs and PFSs to address difficulties from daily life more precisely and reliably. Decision-making problems often involve uncertainty, fuzziness, and imprecision, which require advanced mathematical tools to handle. Traditional FS theory provides a framework to manage such issues, but it is often limited in its ability to capture complex uncertainties. To address this, various extensions of FSs have been proposed, such as IFSs, PFSs, and q-ROFSs. Among these, q-ROFS is particularly useful because it offers a flexible and generalized structure, allowing for a wider range of uncertainty representation compared to IFS and PFS.

The intensity of similarity between two sets is assessed using the similarity metric. Many scholars have suggested various techniques in the literature to compare the similarities of two sets or items in a wide range of contexts, including categorizing and data analysis, pattern recognition, and multi-criteria decision-making (MCDM) applications [14]. The TOPSIS was conceptualized by Hussain & Yang [15] with application to MCDM and clustering using the Hausdorff metric. Jaccard index of IFSs and its application to clustering is proposed by Hwang *et al.* [16]. PFS entropy for solving MCDM problems and pattern recognition is proposed by Yang & Hussain [17]. Hussain *et al.* [18] proposed new similarity measures for PFSs to solve MCDM problems. The fuzzy entropy measure is a sufficient tool with a wide range of applications [19] in medical diagnosis.

While various similarity measures for PFSs have been extensively explored in the literature, there remains a significant gap in the application of similarity measures based on the Jaccard index for q-ROFSs. The Jaccard index, renowned for its effectiveness in cluster analysis and similarity assessment, has not yet been sufficiently integrated into the framework of q-ROFSs. This gap presents a compelling opportunity to develop and investigate new similarity measures that leverage the strengths of the Jaccard index. By addressing this gap, we can enhance the analytical capabilities of q-ROFSs, providing more accurate and reliable tools for decision-making and classification scenarios.

Hence, we address this gap by proposing a novel similarity measure for q-ROFSs based on the Jaccard Index in this paper. We provide several examples to demonstrate the effectiveness and applicability of our proposed method. To further substantiate our approach, we use numerical examples to compare our suggested similarity measure with existing ones, revealing its superior performance in various scenarios. Additionally, we extend our work by developing an Orthopairian TOPSIS (OTOPSIS) method that incorporates the proposed Jaccard similarity measures for q-ROFSs. This improvement not only fills the existing gap but also enhances decision-making processes by offering a more refined and effective tool for similarity assessment in MCDM problems.

The major contribution of this research is outlined as follows:

- i. introduction of a novel similarity measure based on the Jaccard index specifically for q-ROFSs;
- ii. validation with numerical examples to demonstrate the reliability and effectiveness;
- iii. conduction of comparative analysis with existing measures to show its superiority;

- iv. development of OTOPSIS based on the proposed similarity measure;
- v. real-life MCDM application to demonstrate the practical applicability and suitability of the proposed schemes.

The organization of the article is as follows: fundamental definitions are presented in Section 2. Constructing similarity measures for q-ROFSs based on the Jaccard index is the focus of Section 3. Section 4 contains a numerical simulation study with examples to show how our suggested similarity measures make sense. Furthermore, a numerical comparison and a solution to a pattern recognition issue are also provided to reveal the advantages of our suggested strategy over the ones already in use. Section 5 describes how the suggested similarity measures can be used in MCDM by establishing OTOPSIS. Moreover, the proposed OTOPSIS is implemented to solve a cancer diagnosis problem in Section 6. The results of the analysis and concluding remarks are presented in Section 7.

2. Preliminaries

Here, we briefly discuss the basic ideas of FSs, IFSs, PFSs, and q-ROFSs as well as the most used similarity measure.

Definition 1 [20]. A FS $\hat{K}$ in $\hat{X}$ is written as $\hat{K} = \{\hat{x}, \mu_{\hat{K}}(\hat{x}) | \hat{x} \in \hat{X}\}$. Here, $\mu_{\hat{K}}(\hat{x}) : \hat{X} \rightarrow [0,1]$ represents the belongingness function of $\hat{K}$.

Definition 2 [7]. An IFS $\hat{L}$ in the universe of discourse $\hat{X}$ is written as $\hat{L} = \{\hat{x}, \mu_{\hat{L}}(\hat{x}), \nu_{\hat{L}}(\hat{x}) | \hat{x} \in \hat{X}\}$. Here, $\mu_{\hat{L}}(\hat{x}) : \hat{X} \rightarrow [0,1]$ indicates the intensity of belongingness and $\nu_{\hat{L}}(\hat{x}) : \hat{X} \rightarrow [0,1]$ is the intensity of non-belongingness, with $0 \leq \mu_{\hat{L}}(\hat{x}) + \nu_{\hat{L}}(\hat{x}) \leq 1$. The degree of hesitancy is $\pi_{\hat{L}}(\hat{x}) : \hat{X} \rightarrow [0,1]$ and $\pi_{\hat{L}}(\hat{x}) = 1 - (\mu_{\hat{L}}(\hat{x}) + \nu_{\hat{L}}(\hat{x}))$.

Definition 3 [12]. Suppose $\hat{X}$ be a universal set. A PFS $\hat{M}$ in $\hat{X}$ is represented as $\hat{M} = \{\langle \hat{x}, \mu_{\hat{M}}(\hat{x}), \nu_{\hat{M}}(\hat{x}) \rangle | \hat{x} \in \hat{X}\}$. Here, $\mu_{\hat{M}} : \hat{X} \rightarrow [0,1]$ indicates the belongingness function and $\nu_{\hat{M}} : \hat{X} \rightarrow [0,1]$ is the intensity of non-belongingness function with $0 \leq (\mu_{\hat{M}}(\hat{x}))^2 + (\nu_{\hat{M}}(\hat{x}))^2 \leq 1$. The intensity of hesitancy will be $\pi_{\hat{M}}(\hat{x}) = \sqrt{1 - ((\mu_{\hat{M}}(\hat{x}))^2 + (\nu_{\hat{M}}(\hat{x}))^2)}$.

Definition 4 [12]. Let $\hat{X}$ be the universal set. A q-ROFS $\hat{N}$ is written as $\hat{N} = \{\langle \hat{x}, \mu_{\hat{N}}(\hat{x}), \nu_{\hat{N}}(\hat{x}) \rangle | \hat{x} \in \hat{X}\}$. Here, $\mu_{\hat{N}}(\hat{x}) : \hat{X} \rightarrow [0,1]$ indicate the belongingness function and $\nu_{\hat{N}}(\hat{x}) : \hat{X} \rightarrow [0,1]$ is the non-belongingness function with $0 \leq (\mu_{\hat{N}}(\hat{x}))^q + (\nu_{\hat{N}}(\hat{x}))^q \leq 1$ and the intensity of hesitancy is $\pi_{\hat{N}}(\hat{x}) = \left(1 - (\mu_{\hat{N}}(\hat{x}))^q - (\nu_{\hat{N}}(\hat{x}))^q\right)^{\frac{1}{q}}$.

Definition 5 [21]. If $\hat{K}$ and $\hat{L}$ are two q-ROFSs, then:

- i. $\hat{K} \subseteq \hat{L}$, iff $\forall \hat{x} \in \hat{X}$, $\mu_{\hat{K}}^q(\hat{x}) \leq \mu_{\hat{L}}^q(\hat{x})$ and $\nu_{\hat{K}}^q(\hat{x}) \geq \nu_{\hat{L}}^q(\hat{x})$;
- ii. $\hat{K} = \hat{L}$, iff $\forall \hat{x} \in \hat{X}$, $\mu_{\hat{K}}^q(\hat{x}) = \mu_{\hat{L}}^q(\hat{x})$ and $\nu_{\hat{K}}^q(\hat{x}) = \nu_{\hat{L}}^q(\hat{x})$;
- iii. $\hat{K} \cup \hat{L} = \{\max(\mu_{\hat{K}}(\hat{x}), \mu_{\hat{L}}(\hat{x})), \min(\nu_{\hat{K}}(\hat{x}), \nu_{\hat{L}}(\hat{x}))\} \forall \hat{x} \in \hat{X}$;
- iv. $\hat{K} \cap \hat{L} = \{\min(\mu_{\hat{K}}(\hat{x}), \mu_{\hat{L}}(\hat{x})), \max(\nu_{\hat{K}}(\hat{x}), \nu_{\hat{L}}(\hat{x}))\} \forall \hat{x} \in \hat{X}$;

$$v. \quad \hat{K}^c = \left\{ \langle \hat{x}, \nu_{\hat{K}}(\hat{x}), \mu_{\hat{L}}(\hat{x}) \rangle \right\}, \forall \hat{x} \in \hat{X}.$$

Definition 6 [21]. Let $\hat{K}$, $\hat{L}$, and $\hat{M}$ be any three q-ROFSs in the universal set $\hat{X}$. A similarity measure $S(\hat{K}, \hat{L})$ is defined as a mapping $S: q\text{-ROFS}(\hat{X}) \times q\text{-ROFS}(\hat{X}) \rightarrow [0, 1]$ possess the below axioms:

- i. $0 \leq S(\hat{K}, \hat{L}) \leq 1$;
- ii. $S(\hat{K}, \hat{L}) = 1$ iff $\hat{K} = \hat{L}$;
- iii. $S(\hat{L}, \hat{K}) = S(\hat{K}, \hat{L})$;
- iv. If $K \subseteq L \subseteq M$, then $S(\hat{K}, \hat{L}) \geq S(\hat{K}, \hat{M})$ and $S(\hat{L}, \hat{M}) \geq S(\hat{K}, \hat{M})$.

Recently, Peng & Liu [21] suggested a similarity measure for two q-ROFSs $\hat{K}$ and $\hat{L}$ as follows:

$$S_{\hat{K}}(\hat{K}, \hat{L}) = 1 - \frac{1}{2|\hat{X}|} \sum_{\hat{x} \in \hat{X}} \left| \mu_{\hat{K}}^q(\hat{x}) - \mu_{\hat{L}}^q(\hat{x}) - (\nu_{\hat{K}}^q(\hat{x}) - \nu_{\hat{L}}^q(\hat{x})) \right| \quad (1)$$

3. Proposed Novel Similarity Measure

Jaccard index is a well-known similarity measure that evaluates the similarity between two sets by comparing their intersection and union. For q-ROFS, we extend this idea to measure the similarity between two q-ROFSs. J-index is used as the foundation for developing similarities between two IFSS [16]. Suppose $\hat{K}$ and $\hat{L}$ be two sample sets. The J-index $(\hat{K}, \hat{L})$ is defined as the dimensions of intersection of $\hat{K}$ and $\hat{L}$, which divides by unifications of $\hat{K}$ and $\hat{L}$. If we assume $\hat{Y}_a$ and $\hat{Y}_b$ having n attributes which are either 0 or 1, then the overlapping degree of $\hat{Y}_a$ and $\hat{Y}_b$ can be found by the Jaccard coefficient $J(\hat{Y}_a, \hat{Y}_b)$. It can be find as the ratio of $\hat{Y}_a$ and $\hat{Y}_b$ to the number possessed by $\hat{Y}_a$ and $\hat{Y}_b$, and represented as $J(\hat{Y}_a, \hat{Y}_b) = (\hat{Y}_a \times \hat{Y}_b) / (\hat{Y}_a \times \hat{Y}_a + \hat{Y}_b \times \hat{Y}_b - \hat{Y}_a \times \hat{Y}_b)$.

For example, if $\hat{Y}_a$ and $\hat{Y}_b$ are given as $\hat{Y}_a(0, 1, 1, 0)$ and $\hat{Y}_b(1, 1, 0, 1)$, then their cardinality of intersection and union will be $\hat{Y}_a \times \hat{Y}_b = (0 + 1 + 0 + 0) = 1$ and $\hat{Y}_a \times \hat{Y}_a + \hat{Y}_b \times \hat{Y}_b - \hat{Y}_a \times \hat{Y}_b = (2 + 3 - 1) = 4$, respectively. They provide the dual Jaccard coefficient as $J(\hat{Y}_a, \hat{Y}_b) = 1/4$.

Suppose $\hat{X} = \{\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n\}$ is a discrete universal set. We show the q-ROFSs $\hat{K} = \{ \langle \hat{x}_i, \mu_{\hat{K}}^q(\hat{x}_i), \nu_{\hat{K}}^q(\hat{x}_i) \rangle : \hat{x}_i \in \hat{Y} \}$ and $\hat{L} = \{ \langle \hat{x}_i, \mu_{\hat{L}}^q(\hat{x}_i), \nu_{\hat{L}}^q(\hat{x}_i) \rangle : \hat{x}_i \in \hat{Y} \}$ by utilizing the fuzzy Jaccard coefficient as $J(\mu_{\hat{K}}^q(\hat{x}_i), \mu_{\hat{L}}^q(\hat{x}_i)) = \frac{\mu_{\hat{K}}^q(\hat{x}_i) \times \mu_{\hat{L}}^q(\hat{x}_i)}{(\mu_{\hat{K}}^q(\hat{x}_i))^2 + (\mu_{\hat{L}}^q(\hat{x}_i))^2 - \mu_{\hat{K}}^q(\hat{x}_i) \times \mu_{\hat{L}}^q(\hat{x}_i)}$. If $\mu_{\hat{K}}^q(\hat{x}_i) \neq 0$ and $\mu_{\hat{L}}^q(\hat{x}_i) \neq 0$ but $J(\mu_{\hat{K}}^q(\hat{x}_i), \mu_{\hat{L}}^q(\hat{x}_i)) = 1$. Likewise, we can also take into consider the term $1 - \nu_{\hat{K}}^q(\hat{x}_i)$, $1 - \nu_{\hat{L}}^q(\hat{x}_i)$, $(1 + \mu_{\hat{K}}^q(\hat{x}_i) - \nu_{\hat{K}}^q(\hat{x}_i))$, and $(1 + \mu_{\hat{L}}^q(\hat{x}_i) - \nu_{\hat{L}}^q(\hat{x}_i))$, respectively. Thus, the novel SM between two q-ROFSs $\hat{K}$ and $\hat{L}$ based on the Jaccard index is proposed as follows:

$$s(\hat{K}, \hat{L}) = \frac{1}{3n} \sum_{i=1}^n \left(g(\mu_{\hat{K}}^q(\hat{x}_i), \mu_{\hat{L}}^q(\hat{x}_i)) + g\left(\frac{1-\nu_{\hat{K}}^q(\hat{x}_i), (1-\nu_{\hat{L}}^q(\hat{x}_i))}{+g(1+\mu_{\hat{K}}^q(\hat{x}_i)-\nu_{\hat{K}}^q(\hat{x}_i), (1+\mu_{\hat{L}}^q(\hat{x}_i)-\nu_{\hat{L}}^q(\hat{x}_i))}\right) \right) \quad (2)$$

where $g(s^q, t^q) = 1$ if $s^q = t^q = 0$ and $g(s^q, t^q) = \frac{s^q \times t^q}{(s^q)^2 + (t^q)^2 - s^q \times t^q}$ if $s^q > 0$ and $t^q > 0$.

Now, we show that $S(\hat{K}, \hat{L})$ is a similarity measure between the q-ROFs $\hat{K}$ and $\hat{L}$ as defined by Eq. (1).

Proposition 1. $S(\hat{K}, \hat{L})$ is a similarity between the q-ROFSs $\hat{K}$ and $\hat{L}$.

Proof of Proposition 1. The first condition, we proved since $(s^q - t^q)^2 \geq 0$ and $s^q \geq 0, t^q \geq 0$, we have $(s^q)^2 + (t^q)^2 - s^q \times t^q \geq s^q \times t^q \geq 0$. This implies $0 \leq \frac{s^q \times t^q}{(s^q)^2 + (t^q)^2 - s^q \times t^q} \leq 1$, so $0 \leq g(s^q, t^q) \leq 1$.

All terms of $\mu_{\hat{K}}^q(\hat{x}_i), 1-\nu_{\hat{K}}^q(\hat{x}_i)$ and $(1+\mu_{\hat{K}}^q(\hat{x}_i)-\nu_{\hat{K}}^q(\hat{x}_i))$ are not less than 0, such that $S(\hat{K}, \hat{L})$ is also between 0 and 1. Therefore, $0 \leq S(\hat{K}, \hat{L}) \leq 1$ and the first condition is satisfied.

Then, we prove the second condition. If $\hat{K} = \hat{L}$, we have $\mu_{\hat{K}}^q(\hat{x}) = \mu_{\hat{L}}^q(\hat{x})$ and $\nu_{\hat{K}}^q(\hat{x}) = \nu_{\hat{L}}^q(\hat{x})$.

$$\text{Hence, } g(\mu_{\hat{K}}^q(\hat{x}_i), \mu_{\hat{L}}^q(\hat{x}_i)) = \frac{\mu_{\hat{K}}^q(\hat{x}_i) \times \mu_{\hat{L}}^q(\hat{x}_i)}{(\mu_{\hat{K}}^q(\hat{x}_i))^2 + (\mu_{\hat{L}}^q(\hat{x}_i))^2 - \mu_{\hat{K}}^q(\hat{x}_i) \times \mu_{\hat{L}}^q(\hat{x}_i)} = 1.$$

Similarly, $g(1-\nu_{\hat{K}}^q(\hat{x}_i), 1-\nu_{\hat{L}}^q(\hat{x}_i)) = 1$ and $g(1+\mu_{\hat{K}}^q(\hat{x}_i)-\nu_{\hat{K}}^q(\hat{x}_i), 1+\mu_{\hat{L}}^q(\hat{x}_i)-\nu_{\hat{L}}^q(\hat{x}_i)) = 1$. Therefore,

$$\text{we have } S(\hat{K}, \hat{L}) = \frac{1}{3n} \sum_{i=1}^n \left(g(\mu_{\hat{K}}^q(\hat{x}_i), \mu_{\hat{L}}^q(\hat{x}_i)) + g(1-\nu_{\hat{K}}^q(\hat{x}_i), 1-\nu_{\hat{L}}^q(\hat{x}_i)) + g(1+\mu_{\hat{K}}^q(\hat{x}_i)-\nu_{\hat{K}}^q(\hat{x}_i), 1+\mu_{\hat{L}}^q(\hat{x}_i)-\nu_{\hat{L}}^q(\hat{x}_i)) \right) = \frac{1}{3n} \sum_{i=1}^n (1+1+1) = 1.$$

If $S(\hat{K}, \hat{L}) = 1$, we have $g(\mu_{\hat{K}}^q(\hat{x}_i), \mu_{\hat{L}}^q(\hat{x}_i)) = 1$ and $g(1-\nu_{\hat{K}}^q(\hat{x}_i), 1-\nu_{\hat{L}}^q(\hat{x}_i)) = 1$ since $g(\mu_{\hat{K}}^q(\hat{x}_i), \mu_{\hat{L}}^q(\hat{x}_i))$ if implies $\frac{\mu_{\hat{K}}^q(\hat{x}_i) \times \mu_{\hat{L}}^q(\hat{x}_i)}{(\mu_{\hat{K}}^q(\hat{x}_i))^2 + (\mu_{\hat{L}}^q(\hat{x}_i))^2 - \mu_{\hat{K}}^q(\hat{x}_i) \times \mu_{\hat{L}}^q(\hat{x}_i)} = 1$. So we have that

$(\mu_{\hat{K}}^q(\hat{x}_i) - \mu_{\hat{L}}^q(\hat{x}_i))^2 = 0$. Then, $\mu_{\hat{K}}^q(\hat{x}_i) = \mu_{\hat{L}}^q(\hat{x}_i)$. Similarly, using $g(1-\nu_{\hat{K}}^q(\hat{x}_i), 1-\nu_{\hat{L}}^q(\hat{x}_i)) = 1$ we have that $\nu_{\hat{K}}^q(\hat{x}_i) = \nu_{\hat{L}}^q(\hat{x}_i)$. Hence $\hat{K} = \hat{L}$ and the second condition is fulfilled.

Since, $g(s, t) = g(t, s)$ so $S(\hat{K}, \hat{L}) = S(\hat{L}, \hat{K})$ is easy to solve, which is the third condition.

To prove the fourth condition, considering h be a function as $h(\hat{y}) = \frac{\hat{x}\hat{y}}{\hat{x}^2\hat{y}^2 - \hat{x}\hat{y}}$,

$$\frac{d}{d\hat{y}} h(\hat{y}) = \frac{\hat{x}(\hat{x} - \hat{y})}{(\hat{x}^2 + \hat{y}^2 - \hat{x}\hat{y})^2}. \text{ Here, if } 0 < \hat{x} < \hat{y}, \text{ then } h(\hat{y}) \text{ must be decreasing. Also, if } 0 < \hat{y} < \hat{x}, \text{ then}$$

$f(\hat{y})$ will be an increasing function of $\hat{y}$.

If $\hat{K} \subset \hat{L} \subset \hat{M}$, then for each $\hat{x} \in \hat{y}$ we have $\mu_{\hat{K}}^q(\hat{x}) \leq \mu_{\hat{L}}^q(\hat{x}) \leq \mu_{\hat{M}}^q(\hat{x})$ and $\nu_{\hat{K}}^q(\hat{x}) \geq \nu_{\hat{L}}^q(\hat{x}) \geq \nu_{\hat{M}}^q(\hat{x})$. This implies:

$$g(\mu_{\hat{K}}^q(\hat{x}_i), \mu_{\hat{M}}^q(\hat{x}_i)) = \frac{\mu_{\hat{K}}^q(\hat{x}_i) \cdot \mu_{\hat{M}}^q(\hat{x}_i)}{(\mu_{\hat{K}}^q(\hat{x}_i))^2 + (\mu_{\hat{M}}^q(\hat{x}_i))^2 - \mu_{\hat{K}}^q(\hat{x}_i) \times \mu_{\hat{M}}^q(\hat{x}_i)} \leq \frac{\mu_{\hat{K}}^q(\hat{x}_i) \cdot \mu_{\hat{L}}^q(\hat{x}_i)}{(\mu_{\hat{K}}^q(\hat{x}_i))^2 + (\mu_{\hat{L}}^q(\hat{x}_i))^2 - \mu_{\hat{K}}^q(\hat{x}_i) \times \mu_{\hat{L}}^q(\hat{x}_i)} \\ = g(\mu_{\hat{K}}^q(\hat{x}_i), \mu_{\hat{L}}^q(\hat{x}_i)).$$

$$g(1 - \nu_{\hat{K}}^q(\hat{x}_i), 1 - \nu_{\hat{M}}^q(\hat{x}_i)) = \frac{(1 - \nu_{\hat{K}}^q(\hat{x}_i))(1 - \nu_{\hat{M}}^q(\hat{x}_i))}{(1 - \nu_{\hat{K}}^q(\hat{x}_i))^2 + (1 - \nu_{\hat{M}}^q(\hat{x}_i))^2 - (1 - \nu_{\hat{K}}^q(\hat{x}_i))(1 - \nu_{\hat{M}}^q(\hat{x}_i))} \\ \leq \frac{(1 - \nu_{\hat{K}}^q(\hat{x}_i))(1 - \nu_{\hat{L}}^q(\hat{x}_i))}{(1 - \nu_{\hat{K}}^q(\hat{x}_i))^2 + (1 - \nu_{\hat{L}}^q(\hat{x}_i))^2 - (1 - \nu_{\hat{K}}^q(\hat{x}_i))(1 - \nu_{\hat{L}}^q(\hat{x}_i))} = g((1 - \nu_{\hat{K}}^q(\hat{x}_i))(1 - \nu_{\hat{L}}^q(\hat{x}_i))).$$

$$g((1 + \mu_{\hat{K}}^q(\hat{x}_i) - \nu_{\hat{K}}^q(\hat{x}_i)), (1 + \mu_{\hat{M}}^q(\hat{x}_i) - \nu_{\hat{M}}^q(\hat{x}_i))) \\ = \frac{(1 + \mu_{\hat{K}}^q(\hat{x}_i) - \nu_{\hat{K}}^q(\hat{x}_i)) \times (1 + \mu_{\hat{M}}^q(\hat{x}_i) - \nu_{\hat{M}}^q(\hat{x}_i))}{(1 + \mu_{\hat{K}}^q(\hat{x}_i) - \nu_{\hat{K}}^q(\hat{x}_i))^2 + (1 + \mu_{\hat{M}}^q(\hat{x}_i) - \nu_{\hat{M}}^q(\hat{x}_i))^2 - (1 + \mu_{\hat{K}}^q(\hat{x}_i) - \nu_{\hat{K}}^q(\hat{x}_i)) \times (1 + \mu_{\hat{M}}^q(\hat{x}_i) - \nu_{\hat{M}}^q(\hat{x}_i))} \\ \leq \frac{(1 + \mu_{\hat{K}}^q(\hat{x}_i) - \nu_{\hat{K}}^q(\hat{x}_i)) \times (1 + \mu_{\hat{L}}^q(\hat{x}_i) - \nu_{\hat{L}}^q(\hat{x}_i))}{(1 + \mu_{\hat{K}}^q(\hat{x}_i) - \nu_{\hat{K}}^q(\hat{x}_i))^2 + (1 + \mu_{\hat{L}}^q(\hat{x}_i) - \nu_{\hat{L}}^q(\hat{x}_i))^2 - (1 + \mu_{\hat{K}}^q(\hat{x}_i) - \nu_{\hat{K}}^q(\hat{x}_i)) \times (1 + \mu_{\hat{L}}^q(\hat{x}_i) - \nu_{\hat{L}}^q(\hat{x}_i))} \\ = g(1 + \mu_{\hat{K}}^q(\hat{x}_i) - \nu_{\hat{K}}^q(\hat{x}_i), 1 + \mu_{\hat{L}}^q(\hat{x}_i) - \nu_{\hat{L}}^q(\hat{x}_i)).$$

Thus, we have $S(\hat{K}, \hat{M}) \leq S(\hat{K}, \hat{L})$. Similarly, we can also show that $S(\hat{K}, \hat{M}) \leq S(\hat{N}, \hat{L})$. So fourth condition is fulfilled.

A generalized similarity measure of q-ROFs $\hat{K}$ and $\hat{L}$ on a function f is:

$$s(K, L) = \frac{1}{3n} \sum_{i=1}^n \left(g(\mu_{\hat{K}}^q(\hat{x}_i), \mu_{\hat{L}}^q(\hat{x}_i)) + g(1 - \nu_{\hat{K}}^q(\hat{x}_i), 1 - \nu_{\hat{L}}^q(\hat{x}_i)) + g(1 + \mu_{\hat{K}}^q(\hat{x}_i) - \nu_{\hat{K}}^q(\hat{x}_i), 1 + \mu_{\hat{L}}^q(\hat{x}_i) - \nu_{\hat{L}}^q(\hat{x}_i)) \right) \quad (3)$$

$$\text{where } f \text{ is a given function and } g(s^q, t^q) = \begin{cases} 1 & \text{if } s^q = t^q = 0 \\ \frac{s^q \times t^q}{(s^q)^2 + (t^q)^2 - s^q \times t^q} & \text{if } s^q > 0 \text{ and } t^q > 0 \end{cases}$$

Proposition 2. From Eq. (2), the function f put with $\frac{d}{d\hat{x}} f(\hat{x}) > 0$ and $f(0) = 0$, then the similarity measure $S(\hat{K}, \hat{L})$ is a similarity measure between q-ROFs $\hat{K}$ and $\hat{L}$.

Proof of Proposition 2. Eq. (2), the function f put with $\frac{d}{d\hat{x}} f(\hat{x}) > 0$ is monotonically increasing. It shows that $f(0) = 0$ such that $f(\hat{y}) \geq 0$ for $\hat{y} \geq 0$. In this case, Eq. (2), give similar results like Eq. (1) and that condition replace only s and t with $f(\hat{s})$ and $f(\hat{t})$. Thus, we followed Proof of Proposition 1.

Suppose the function $f(\hat{x}) = 1 - e^{-\hat{x}}$. Then, a novel similarity will be extracted as:

$$\begin{aligned}
 S_1(\hat{K}, \hat{L}) = & \frac{1}{3n} \sum_{i=1}^n \frac{\left(1 - \exp^{-\mu_K^q(\hat{x}_i)}\right) \left(1 - \exp^{-\mu_L^q(\hat{x}_i)}\right)}{\left(1 - \exp^{-\mu_K^q(\hat{x}_i)}\right)^2 + \left(1 - \exp^{-\mu_L^q(\hat{x}_i)}\right)^2 - \left(1 - \exp^{-\mu_K^q(\hat{x}_i)}\right) \left(1 - \exp^{-\mu_L^q(\hat{x}_i)}\right)} \\
 & + \frac{\left(1 - \exp^{-(1-\nu_K^q(\hat{x}_i))}\right) \left(1 - \exp^{-(1-\nu_L^q(\hat{x}_i))}\right)}{\left(1 - \exp^{-(1-\nu_K^q(\hat{x}_i))}\right)^2 + \left(1 - \exp^{-(1-\nu_L^q(\hat{x}_i))}\right)^2 - \left(1 - \exp^{-(1-\nu_K^q(\hat{x}_i))}\right) \left(1 - \exp^{-(1-\nu_L^q(\hat{x}_i))}\right)} \\
 & + \frac{\left(1 - \exp^{-(1+\mu_K^q(\hat{x}_i)-\nu_K^q(\hat{x}_i))}\right) \left(1 - \exp^{-(1+\mu_L^q(\hat{x}_i)-\nu_L^q(\hat{x}_i))}\right)}{\left(1 - \exp^{-(1+\mu_K^q(\hat{x}_i)-\nu_K^q(\hat{x}_i))}\right)^2 + \left(1 - \exp^{-(1+\mu_L^q(\hat{x}_i)-\nu_L^q(\hat{x}_i))}\right)^2 - \left(1 - \exp^{-(1+\mu_K^q(\hat{x}_i)-\nu_K^q(\hat{x}_i))}\right) \left(1 - \exp^{-(1+\mu_L^q(\hat{x}_i)-\nu_L^q(\hat{x}_i))}\right)}
 \end{aligned} \quad (4)$$

Another extension similarity measures between q-ROFSs $\hat{K}$ and $\hat{L}$ is suggested as:

$$S_f(\hat{K}, \hat{L}) = \frac{1}{3n} \sum_{i=1}^n \left(\frac{g(\mu_K^q(\hat{x}_i), \mu_L^q(\hat{x}_i)) + g(1 - \nu_K^q(\hat{x}_i), 1 - \nu_L^q(\hat{x}_i))}{g(1 + \mu_K^q(\hat{x}_i) - \nu_K^q(\hat{x}_i), 1 + \mu_L^q(\hat{x}_i) - \nu_L^q(\hat{x}_i))} \right) \quad (5)$$

where f is a given function and $g(s^q, t^q) = \begin{cases} 1, & \text{if } s^q = t^q = 0 \\ \frac{f(s^q)f(t^q)}{(f(s^q))^2 + (f(t^q))^2 - f(s^q)f(t^q)}, & \text{if } s^q > 0 \text{ and } t^q > 0 \end{cases}$

Proposition 3. If the function f in Eq. (3) having $\frac{d}{d\hat{x}} f(\hat{x}) < 0$ and $f(1) = 0$, then $S_f(\hat{K}, \hat{L})$ is a similarity measure of q-ROFSs q-ROFs $\hat{K}$ and $\hat{L}$.

Proof of Proposition 3. Eq. (3) suits with $\frac{d}{d\hat{x}} f(\hat{x}) < 0$, f is the with monotonous function that decreases $\hat{x}$. Since, $f(1) = 0$ such that $f(\hat{y}) \geq 0$ for all $\hat{y} \geq 0$. Hence, Proposition 3 is similar to Proposition 1.

Suppose the function $f(\hat{x}) = \exp^{-\hat{x}} - \exp^{-1}$. We get one more similarity measure $S_2(\hat{K}, \hat{L})$ as:

$$\begin{aligned}
 S_2(\hat{K}, \hat{L}) = & \frac{1}{3n} \sum_{i=1}^n \frac{\left(\exp^{-\mu_K^q(\hat{x}_i)} - \exp^{-1}\right) \left(\exp^{-\mu_L^q(\hat{x}_i)} - \exp^{-1}\right)}{\left(\exp^{-\mu_K^q(\hat{x}_i)} - \exp^{-1}\right)^2 + \left(\exp^{-\mu_L^q(\hat{x}_i)} - \exp^{-1}\right)^2 - \left(\exp^{-\mu_K^q(\hat{x}_i)} - \exp^{-1}\right) \left(\exp^{-\mu_L^q(\hat{x}_i)} - \exp^{-1}\right)} \\
 & + \frac{\left(\exp^{-(1-\nu_K^q(\hat{x}_i))} - \exp^{-1}\right) \left(\exp^{-(1-\nu_L^q(\hat{x}_i))} - \exp^{-1}\right)}{\left(\exp^{-(1-\nu_K^q(\hat{x}_i))} - \exp^{-1}\right)^2 + \left(\exp^{-(1-\nu_L^q(\hat{x}_i))} - \exp^{-1}\right)^2 - \left(\exp^{-(1-\nu_K^q(\hat{x}_i))} - \exp^{-1}\right) \left(\exp^{-(1-\nu_L^q(\hat{x}_i))} - \exp^{-1}\right)} \\
 & + \frac{\left(\exp^{-(1+\mu_K^q(\hat{x}_i)-\nu_K^q(\hat{x}_i))} - \exp^{-1}\right) \left(\exp^{-(1+\mu_L^q(\hat{x}_i)-\nu_L^q(\hat{x}_i))} - \exp^{-1}\right)}{\left(\exp^{-(1+\mu_K^q(\hat{x}_i)-\nu_K^q(\hat{x}_i))} - \exp^{-1}\right)^2 + \left(\exp^{-(1+\mu_L^q(\hat{x}_i)-\nu_L^q(\hat{x}_i))} - \exp^{-1}\right)^2 - \left(\exp^{-(1+\mu_K^q(\hat{x}_i)-\nu_K^q(\hat{x}_i))} - \exp^{-1}\right) \left(\exp^{-(1+\mu_L^q(\hat{x}_i)-\nu_L^q(\hat{x}_i))} - \exp^{-1}\right)}
 \end{aligned} \quad (6)$$

4. Application to Pattern Recognition with Comparisons

We give some numerical examples to validate our suggested similarity measures. The following examples are presented to compare our suggested similarity measures outlined in Eq. (2), Eq. (4) and Eq. (6) with the existing similarity measure defined in Eq. (1) in the context of pattern recognition. The study of how machines can recognize various patterns is known as pattern recognition which categorizes various patterns according to their class.

Example 1. Let us consider that there are three q-ROFS patterns in $\hat{X} = \{\hat{x}_1, \hat{x}_2, \hat{x}_3\}$. These patterns are indicated by the following q-ROFS: $\hat{K}_1 = \{(\hat{x}_1, 0.2, 0.2), (\hat{x}_2, 0.3, 0.3), (\hat{x}_3, 0.4, 0.4)\}$, $\hat{K}_2 = \{(\hat{x}_1, 0.3, 0.3), (\hat{x}_2, 0.3, 0.3), (\hat{x}_3, 0.3, 0.3)\}$, and $\hat{K}_3 = \{(\hat{x}_1, 0.2, 0.2), (\hat{x}_2, 0.2, 0.2), (\hat{x}_3, 0.2, 0.2)\}$. Suppose a sample $\hat{L} = \{(\hat{x}_1, 0.2, 0.2), (\hat{x}_2, 0.3, 0.3), (\hat{x}_3, 0.4, 0.4)\}$ is given. To identify the belongingness of sample $\hat{L}$ with other patterns, we apply the procedure suggested by Peng & Liu [21] as: $S_k(\hat{K}_1, \hat{L}) = 1.0000$, $S_k(\hat{K}_2, \hat{L}) = 1.0000$, and $S_k(\hat{K}_3, \hat{L}) = 1.0000$. The above results indicate that the procedure of Peng & Liu [21] could not identify the q-ROFS sample. Thus, it can be concluded that the aforementioned procedure is not applicable to recognize q-ROFS patterns. But, our suggested similarity measures outlined in Eq. (2), Eq. (4) and Eq. (6) provide acceptable results which are given as $S(\hat{K}_1, \hat{L}) = 1$, $S(\hat{K}_2, \hat{L}) = 0.8609$, $S(\hat{K}_3, \hat{L}) = 0.8348$, $S_1(\hat{K}_1, \hat{L}) = 1$, $S_1(\hat{K}_2, \hat{L}) = 0.8830$, $S_1(\hat{K}_3, \hat{L}) = 0.8358$, $S_2(\hat{K}_1, \hat{L}) = 1$, $S_2(\hat{K}_2, \hat{L}) = 0.6569$, and $S_2(\hat{K}_3, \hat{L}) = 0.6108$. From the above results, it is clear that pattern $\hat{K}_1$ and sample $\hat{L}$ are exactly same, while the remaining patterns $\hat{K}_2$ and $\hat{K}_3$ are different. The above numerical analysis results of Eq. (2), Eq. (4) and Eq. (6) clearly shows that sample $\hat{L}$ match to the pattern $\hat{K}_1$ according to principle of highest similarity among two q-ROFSs.

Example 2. Consider two q-ROFS patterns in the universe $\hat{X} = \{\hat{x}_1, \hat{x}_2\}$ as $\hat{K}_1 = \{(\hat{x}_1, 0.5, 0.5), (\hat{x}_2, 0.4, 0.4), (\hat{x}_3, 0.3, 0.3)\}$ and $\hat{K}_2 = \{(\hat{x}_1, 0.4, 0.4), (\hat{x}_2, 0.4, 0.4), (\hat{x}_3, 0.4, 0.4)\}$. Suppose the sample is given as $\hat{L} = \{(\hat{x}_1, 0.5, 0.5), (\hat{x}_2, 0.4, 0.4), (\hat{x}_3, 0.3, 0.3)\}$. The results obtained using the similarity method given by Peng & Liu [21] are $S_k(\hat{K}_1, \hat{L}) = 1.0000$ and $S_k(\hat{K}_2, \hat{L}) = 1.0000$. It is noticed that the pattern $\hat{K}_1$ is exactly same as the sample $\hat{L}$. However, the pattern $\hat{K}_2$ is different from sample $\hat{L}$. Intuitively, we expect that the similarity between the pattern $\hat{K}_1$ to the sample $\hat{L}$ should be different from the similarity between the pattern $\hat{K}_2$ to the sample $\hat{L}$. Unfortunately, the method given by Peng & Liu [21] could not show the correct classification. On the other hand, our suggested measures from Eq. (2), Eq. (4) and Eq. (6) give intuitively acceptable results as $S(\hat{K}_1, \hat{L}) = 1.0000$, $S(\hat{K}_2, \hat{L}) = 0.9050$, $S_1(\hat{K}_1, \hat{L}) = 1.0000$, $S_1(\hat{K}_2, \hat{L}) = 0.9206$, $S_2(\hat{K}_1, \hat{L}) = 1.0000$, and $S_2(\hat{K}_2, \hat{L}) = 0.9929$. The above results using our suggested methods from Eq. (2), Eq. (4) and Eq. (6) show that the sample $\hat{L}$ is exactly same to the sample $\hat{K}_1$, which is intuitively appropriate.

5. Application to Multi-Criteria Decision-Making

An important part of decision science is the decision-making process, where we collect related information to make appropriate decisions. In many situations, we hesitate to choose the best one from the many alternatives available. In order to make good decisions about complex everyday problems of any kind, we need a powerful algorithm. Therefore, to solve complex problems in daily

life, including decision-making, we developed an OTOPSIS using our proposed similarity measure developed in Eq. (3) based on the Jaccard index with q-ROFS.

The steps of OTOPSIS are as follows:

Step 1: Constructing orthopairian fuzzy decision matrix. Define the alternatives and criteria. Each alternative is evaluated with respect to each criterion. We indicate the set of the alternatives as $\hat{K} = \{\hat{K}_1, \hat{K}_2, \dots, \hat{K}_m\}$ with the criteria $\hat{L} = \{\hat{L}_1, \hat{L}_2, \dots, \hat{L}_n\}$ by $\hat{K}_i(\hat{L}_j) = P(\mu_{ij}, \nu_{ij})$. Then, the decision matrix can be represented as $D = (\hat{K}_i(\hat{L}_j))_{m \times n}$, where $P = (\mu_{ij}, \nu_{ij})$ represented a q-rung orthopair fuzzy number.

Step 2: Determine the ideal and anti-ideal solutions.

$$P^+ = \left[\left\{ \hat{L}_j, \left\langle \left(\vee \mu_{ij}(\hat{L}_j) : j \in \hat{L} \right), \left(\wedge \mu_{ij}(\hat{L}_j) : j \in \hat{M} \right) \right\rangle, \left\langle \left(\wedge \nu_{ij}(\hat{L}_j) : j \in \hat{L} \right), \left(\vee \nu_{ij}(\hat{L}_j) : j \in \hat{M} \right) \right\rangle : i \in m \right\} \right] \quad (7)$$

$$P^- = \left[\left\{ \hat{L}_j, \left\langle \left(\wedge \mu_{ij}(\hat{L}_j) : j \in \hat{L} \right), \left(\vee \mu_{ij}(\hat{L}_j) : j \in \hat{M} \right) \right\rangle, \left\langle \left(\vee \nu_{ij}(\hat{L}_j) : j \in \hat{L} \right), \left(\wedge \nu_{ij}(\hat{L}_j) : j \in \hat{M} \right) \right\rangle : i \in m \right\} \right] \quad (8)$$

Step 3: Calculating the distances from ideal solutions.

$$\hat{E}^+(\hat{K}_i) = S_{WH}(\hat{K}_i, \hat{P}^+) \quad (9)$$

$$\hat{E}^-(\hat{K}_i) = S_{WH}(\hat{K}_i, \hat{P}^-) \quad (10)$$

Step 4: Calculating the relative closeness.

$$\hat{E}(\hat{K}_i) = \frac{\hat{P}^-(\hat{K}_i)}{(\hat{K}_i^+) + (\hat{K}_i^-)} \quad (10)$$

Step 5: Ranking the alternatives. The alternatives are finally ranked in a preferred order, according to the maximum closeness degree. The maximum value of relative closeness degree is taken as the most favorable alternative.

6. Orthopairian TOPSIS: Application in Cancer Diagnosis

Nowadays, various types of cancer are increasingly prevalent in our society, with skin cancer being one of the most concerning. This serious disease primarily results from exposure to UV radiation, which damages the DNA in skin cells. Skin cancer often develops in areas that are frequently exposed to sunlight, such as the hands, feet, face, and neck. However, it is important to note that skin cancer can also appear in less visible areas of the body. In addition to UV exposure, other risk factors contribute to the development of skin cancer, including genetic predisposition, a weakened immune system, and certain medications that increase sensitivity to sunlight. While skin cancer is primarily associated with UV radiation, it can also spread through indirect means, such as contaminated items like shared razors or syringes, underscoring the importance of proper hygiene and care. As awareness of skin cancer continues to grow, prevention remains a crucial focus. Regular skin examinations, wearing protective clothing, and using sunscreen are vital steps in reducing the risk of skin cancer. Education about the signs and symptoms of skin cancer can lead to earlier

detection and improved outcomes. Fortunately, there are several effective treatment options available for skin cancer, including a range of surgical methods. These methods can be tailored to the specific type and stage of cancer, ensuring that each patient receives appropriate care.

A variety of surgical methods serve as alternatives for treating skin cancer, as $\hat{K}_1$ = mobs surgery, $\hat{K}_2$ = cryosurgery, $\hat{K}_3$ = laser surgery, and $\hat{K}_4$ = radio therapy. For the above given alternatives some criteria to evaluate them are $\hat{M}_1$ = effectiveness, $\hat{M}_2$ = availability, $\hat{M}_3$ = recovery time; and $\hat{M}_4$ = side effects.

The decision matrix is given in Table 1. The alternatives assessment $\hat{K}_i$ below the criteria $\hat{M}_j$ are shown in the decision matrix $D = (\hat{K}_i(\hat{M}_j))_{m \times n}$. The aim is to find the most suitable treatment method using fuzzy OTOPSIS method. The alternatives are given as:

- i. $\hat{K}_1 = (\hat{x}_1, 0.9, 0.6), (\hat{x}_2, 0.7, 0.6), (\hat{x}_3, 0.9, 0.1), (\hat{x}_4, 0.7, 0.7);$
- ii. $\hat{K}_2 = (\hat{x}_1, 0.4, 0.6), (\hat{x}_2, 0.7, 0.6), (\hat{x}_3, 0.8, 0.6), (\hat{x}_4, 0.5, 0.1);$
- iii. $\hat{K}_3 = (\hat{x}_1, 0.5, 0.5), (\hat{x}_2, 0.6, 0.7), (\hat{x}_3, 0.5, 0.3), (\hat{x}_4, 0.6, 0.1);$
- iv. $\hat{K}_4 = (\hat{x}_1, 0.7, 0.1), (\hat{x}_2, 0.9, 0.4), (\hat{x}_3, 0.8, 0.3), (\hat{x}_4, 0.7, 0.6).$

Table 1
q-rung orthopair fuzzy decision matrix

$\hat{K}_i$	$\hat{M}_1$	$\hat{M}_2$	$\hat{M}_3$	$\hat{M}_4$
$\hat{K}_1$	(0.9, 0.6)	(0.7, 0.6)	(0.9, 0.1)	(0.7, 0.7)
$\hat{K}_2$	(0.4, 0.6)	(0.7, 0.6)	(0.8, 0.6)	(0.5, 0.1)
$\hat{K}_3$	(0.5, 0.5)	(0.6, 0.7)	(0.5, 0.3)	(0.6, 0.1)
$\hat{K}_4$	(0.7, 0.1)	(0.9, 0.4)	(0.8, 0.3)	(0.7, 0.6)

Table 1 displays the matrix for making fuzzy decisions. We standardized the decision-making matrix for cost and benefit considerations. The positive (q-ROFPI) and negative ideal solution (q-ROFNIS) are constructed respectively as $\hat{P}^+ = \{(\hat{x}_1, 0.9, 0.1), (\hat{x}_2, 0.9, 0.4), (\hat{x}_3, 0.9, 0.1), (\hat{x}_4, 0.7, 0.1)\}$ and $\hat{P}^- = \{(\hat{x}_1, 0.4, 0.6), (\hat{x}_2, 0.6, 0.7), (\hat{x}_3, 0.5, 0.6), (\hat{x}_4, 0.5, 0.7)\}$. Next, the similarity between each alternative to q-ROFPI and q-ROFNIS was calculated and given in Table 2.

Table 2
Similarity between q-ROFPI and q-ROFNIS

$\hat{K}_i$	$\hat{E}^+(\hat{K}_i)$	$\hat{E}^-(\hat{K}_i)$
$\hat{K}_1$	0.2342	0.1870
$\hat{K}_2$	0.1954	0.2295
$\hat{K}_3$	0.1920	0.2221
$\hat{K}_4$	0.2352	0.2932

The similarity between each alternative to q-ROFPI and q-ROFNIS was used to evaluate the degree of relative closeness as $\hat{E}(\hat{K}_1) = 0.4441$, $\hat{E}(\hat{K}_2) = 0.5401$, $\hat{E}(\hat{K}_3) = 0.5363$, and $\hat{E}(\hat{K}_4) = 0.5549$.

Then, the alternatives were ranked in preferred order as $\hat{K}_4 \succ \hat{K}_2 \succ \hat{K}_3 \succ \hat{K}_1$. The alternative having the maximum closeness degree was taken as the most suitable alternative. Thus, the alternative $\hat{K}_4$ was taken as the most favorable alternative.

7. Conclusion

This paper presented a novel similarity measure for q-ROFSs, which is based on the Jaccard index and offers a significant contribution to decision-making processes under uncertainty. The proposed method creates a robust and versatile framework applicable to MCDM, risk assessment, and clustering problems. Given the frequent application of the Jaccard index in comparing the similarity and diversity of sample sets, the presented approach provides a practical and intuitive tool for evaluating complex scenarios.

Numerical analyzes are performed to demonstrate the utility and applicability of the proposed similarity measure. The results confirm the rationality, effectiveness, and adaptability of the proposed technique in addressing complex decision-making challenges. Future research could look at integrating this similarity measure with other fuzzy techniques to potentially expand its applicability and improve its effectiveness in different domains.

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Conflicts of Interest

The authors declare no conflicts of interest.

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