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A Sustainable Deteriorating Inventory Model with Carbon Emission Reduction Policies

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ABSTRACT

In inventory models, profit can be significantly improved by assuming demand according to the requirements of the market situation. In this study, demand is represented as stock-, time-, or price-dependent to provide a realistic view of market patterns. The production rate is demand-dependent. Time-dependent deterioration is allowed in the system. To address environmental concerns, carbon emissions from inventory holding and replenishment activities are integrated into the decision framework. The model is compared with and without the effect of a trade-credit policy. Numerical examples support the validity of the proposed model. Managerial insights demonstrate the application of the proposed model. Lastly, the study concludes with future research directions.

1. Introduction

Demand analysis has shown that consumer demand is often studied only as price dependent. Stock and price-dependent demand provides a broad understanding of consumer purchasing behaviour, reflecting how both availability and pricing shape demand. However, the modern market is dependent on the availability of stocks. This resulted in a combined model which provides a more realistic framework for thoroughly analyzing fan behaviour and supply strategies. Such models are especially effective in industries where product availability and pricing are vital in influencing customer purchasing decisions.

In payments, a permissible delay is a powerful tool in trade and inventory management that serves as a service of financial support for buyers and a sales promotion apparatus for sellers' prospects. In the current business era, vendors often give buyers a particular time frame to pay their debts after receiving their goods and services, which is called a permissible payment delay. However, while there are advantages for both parties, there are financial implications. A permissible delay in

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payments can significantly improve both profitability and overall business performance when it is managed appropriately.

Inventory management is a crucial, however tolerable difficulty in terms of weakening. Along with decreasing inventory costs, meeting customer needs is important for businesses. But deterioration is one of the principal obstacles in this study. Deterioration refers to the decrease in the value, usability, or value over time of a product or service due to physical and chemical causes. For this reason, the inventory management employs multiple scientific techniques to reduce deterioration, putting into practice proactive approaches to long-term deterioration management, even protecting profitability but also preserving client satisfaction and supply chain sustainability.

An inventory management system is not just about maintaining levels of stock, reducing costs and ensuring timely delivery of the product; however, it also considers controlling the rate of carbon emissions on the environmental impact. Since the emissions of carbon are an important matter in the inventory management systems in the recent era. Greenhouse gases contribute significantly to the activities of purchasing, storing, transporting, and disposing of products. Thus, for operations to be sustainable, inventory management must incorporate techniques to reduce carbon emissions.

Now, the main motivation of the paper can be summarized as:

a) Due to environmental concerns and global carbon regulations, industries are motivated to integrate sustainability and emission control measures into their inventory and production decisions.
b) Many real-life items (such as food, pharmaceuticals, and chemicals) lose value or quality over time, making it essential to model deterioration explicitly in sustainable inventory systems.
c) In competitive markets, product demand often varies with available stock and selling price. Therefore, demand is assumed to be stock and selling price dependent.
d) Trade credit is a common business practice that encourages sales and provides financial flexibility to buyers. Hence, trade-credit policy is also integrated into the model.
e) Carbon emissions during production, storage, and transportation are considered for performing the social and environmental responsibility of the supplier, retailer and buyer.
f) The paper aims to achieve an optimal relation between economic profitability and ecological responsibility by jointly considering cost, pricing, credit period, and emission reduction efforts.
g) The model is a practical framework for an inventory manager to determine optimal policies that improve both sustainability and profitability.

The next portion of the study is arranged as follows: The literature review of this inventory model is presented in Section 2. Section 3 discusses the notations and assumptions used in the study. The model is described in Section 4. Section 5 displays a solution approach to the model. The numerical computation and sensitivity analysis of the evaluated results are provided in Section 6. The managerial insights of the proposed study are described in Section 7. Lastly, the conclusions about the study are provided in Section 8.

2. Literature Review

The literature review of the present study, under different perspectives, is performed in this section.

2.1 Brief survey on stock and price-dependent demand

Price and stock-dependent demand is not a new concept in the inventory model. In order to help merchants optimize earnings by figuring out the best time to replenish inventory and how long it should take, Kaushik [1] presented a model that considers stock, price, and time considerations. Kumar *et al.* [2] derived a system with two warehouses and a partial backlog, with a defined shelf life for deteriorating commodities. The demand rate was regarded as stock dependent, which indicates that demand is influenced by the stock level displayed. A deterministic model with optimizing price and perishable commodity considering both physical deterioration and freshness condition was developed by Agi and Soni [3]. Shah *et al.* [4] studied a predetermined inventory model with a two-warehouse system and partial backlogging by defining shelf life for deteriorating commodities. An integrated sustainable model was formulated by Dey *et al.* [5] to maximize profit while taking environmental concerns into account, reducing discrete setup costs, and maintaining a predictable lead time. Pervin *et al.* [6] developed a model for deteriorating objects, allowing demand to be stock and price dependent and found the impact of credit policy. Mishra [7] considered an inventory diagram for Weibull deterioration with price in addition to stock-dependent demand. Pervin *et al.* [8] inspected the impact of the technique for preservation for a model with price and stock sensitive demand. Stock and price dependent demand make this model more realistic by linking customer behaviour, pricing strategy, and sustainability, and enabling the firm to manage deterioration, credit policy, and carbon emission reduction more effectively.

2.2 Permissible Delay in Payments

Credit policy is crucial for inventory management because it introduces a realistic financial dimension to the inventory system. It connects operational decisions with financial flexibility, which directly influences ordering policies, cash flow, and sustainability outcomes. Uthayakumar *et al.* [9] discovered an inventory diagram of pharmaceutical industries for perishable goods with demand of quadratic and linear holding costs under an allowable credit policy. Gupta *et al.* [10] discussed about perishable products with the influence of credit policy in a two warehouse model system. Pervin *et al.* [11] examine the impact of permissible delay in payment by formulating a model with variable holding costs. Furthermore, Shaikh *et al.* [12] discussed a fuzzy-based model for a perishable product with a credit policy. On the other side, permissible payment delays are an important factor in this research because they integrate the financial realism into the sustainable inventory-based model. Additionally, it permits firms to explore how trade credit, pricing, deterioration and carbon emission policies collectively impact profitability and sustainability, making the study both comprehensive and real-world valuable.

2.3 Deterioration

Deterioration is necessary because it has the ability to capture the realistic character of the majority of physical goods and significantly influences inventory, pricing, and sustainability decisions. Researchers such as Tiwari *et al.* [13] structured a single-seller single-purchaser framework for deteriorating products with defective quality. Rahman *et al.* [14] examined two scenarios in a discount setting while analyzing a diagram for deteriorating products with interval-valued parameters. Das *et al.* [15] presented a deteriorating inventory diagram based on a preservation technique that takes into account multi period-based delay payments. Barman *et al.* [16] formulate an inventory structure model for production with non-instantaneous deteriorating goods.

Additionally, Paul *et al.* [17] examine the impact of default risk factors on cycle time and credit period for a deteriorating model. In this study, deterioration is assumed as a key component because it makes the model realistic, environmentally meaningful, and operationally relevant. It connects inventory losses, pricing, demand, and carbon emissions, helping firms develop sustainable inventory policies that minimize waste and maximize profit.

2.4 Carbon emission reduction

Carbon emission reduction addresses the environmental dimension of the modern inventory model. Among several published authors, Das *et al.* [18] developed a transportation model that accounts for changing carbon emissions. An inventory diagram for degrading products was improved by Yadav *et al.* [19] under green technology investment, taking into account time-sensitive demand, carbon emissions, and selling price. Mashud *et al.* [20] formulated a model that differentiates between order cost decrease and manufactured goods for a green storehouse. Barman *et al.* [21] demonstrated the impact of preservation technology for an Economic Production Quantity (EPQ) framework. Then, Kuo *et al.* [22] investigated the model's carbon footprint of the using carbon footprint standards to determine greenhouse gas emissions statistics. Haque *et al.* [23] formulated a remanufacturing model for defective usable items with green demand. Jiang *et al.* [24] introduced a vendor-buyer model that accounts for carbon emissions. Carbon emission reduction is vital in this article because it transforms the model from a purely economic inventory system into a sustainable decision-making framework. It links profit, environmental responsibility, and operational efficiency, providing comprehensive insights for designing green and competitive inventory policies.

Previous inventory models often considered either emission policies or trade credit separately. This study integrates multiple real-world factors, such as deterioration, sustainable practices, and financial policies, into a unified analytical framework, as shown in Table 1.

Table 1
Comparison of our model with previously published models

Author's	Stock and price dependent demand	Permissible delay in payments	Deterioration	Carbon emission reduction
Agi *et al.* [3]	Yes	-	Yes	-
Barman *et al.* [16]	-	-	Yes	-
Das *et al.* [15]	-	-	-	Yes
Das *et al.* [18]	-	Yes	Yes	-
Dey *et al.* [5]	Yes	-	-	-
Gupta *et al.* [10]	-	Yes	Yes	-
Jiang *et al.* [24]	-	-	-	Yes
Kaushik [1]	Yes	-	Yes	-
Kuo *et al.* [22]	-	-	-	Yes
Kumar *et al.* [2]	Yes	-	-	-
Mashud *et al.* [20]	-	-	Yes	Yes
Mishra [7]	Yes	-	-	-
Rahman *et al.* [14]	-	-	Yes	-
Shah *et al.* [4]	Yes	-	Yes	-
Shaikh *et al.* [12]	-	Yes	Yes	-
Tiwari *et al.* [13]	-	-	Yes	-
Uthayakumar *et al.* [9]	-	Yes	Yes	-
Yadav *et al.* [19]	-	-	-	Yes
This chapter	Yes	Yes	Yes	Yes

Table 1 shows that several authors have considered stock and price dependent demand, permissible delay in payments, product deterioration, and carbon emission reduction separately. But this chapter integrated all the factors in a model and found the impact of all the factors on average cost and optimal credit policy.

3. Notations

All the notations considered for this study are as follows:

Notation	:	Description
$D(p)$	:	Demand function
p	:	Selling price of the item during the inventory cycle
h	:	Holding cost (\$/unit)
θ	:	Deterioration rate
$R(p)$	:	Production rate
S	:	Setup cost (\$)
Q	:	Stock quantity(unit)
$I_1(t)$	:	Inventory level in the production period
$I_2(t)$	:	Inventory level in the non-production period
$I_3(t)$	:	Inventory level in the shortage period
g	:	Green investment
C_t	:	Base carbon tax
z	:	Sensitivity of green investment
c_{eh}	:	Emission rate during holding (\$/unit)
c_{ep}	:	Emission rate during production (\$/unit)
c_{et}	:	Emission rate during transportation (\$/unit)
c	:	Purchasing cost (\$/unit)
I_c	:	Interest charges in stock (\$/unit)
I_e	:	Interest earned in a time (\$/unit)
M	:	The permissible delay period for the supplier
T	:	Cycle length
$TC_1(T)$	:	Average cost for $M \leq t_2$
$TC_2(T)$	:	Average cost for $M > t_2$

Assumptions

The following are the assumptions used in the paper:

1. Demand is price and stock dependent $D(p,\gamma) = \alpha^{\gamma} p^{-\beta}$ where, α is market scale, β is price elasticity $(\beta > 1)$, γ is stock elasticity $(0 \leq \gamma \leq 1)$.
2. We consider green investments g which help to diminish carbon emissions $= C_t e^{-zg}$.
3. Shortages are allowed in the model.
4. Deterioration is time dependent and defined by $\theta = vt$, where v is constant.
5. Production rate is dependent on demand, i.e., $R = a + bD(p,\gamma)$, where a, b are constants.
6. The retailer charges interest with rate I_c when $T \geq M$ and earned interest during the period $t = 0$ to $t = M$ with rate I_e.

4. Model description

The inventory cycle begins at $t = 0$, when the stock level is zero, the production rate R, the deterioration rate is θ, and the demand rate is $D(p)$. The product is produced up until time $t = t_1$. The market has enough demand throughout the time frame $[0, t_1]$ to completely meet customer demand. The inventory level that was accumulated at time t then steadily declines over the course of $\left[t_{1\,,}T\right]$, which raises consumer demand in the marketplace before the stock eventually falls to zero at the time $t = T$. The graphical view of the inventory cycle is shown in Figure 1. Now, the differential equations showing the inventory level are given by

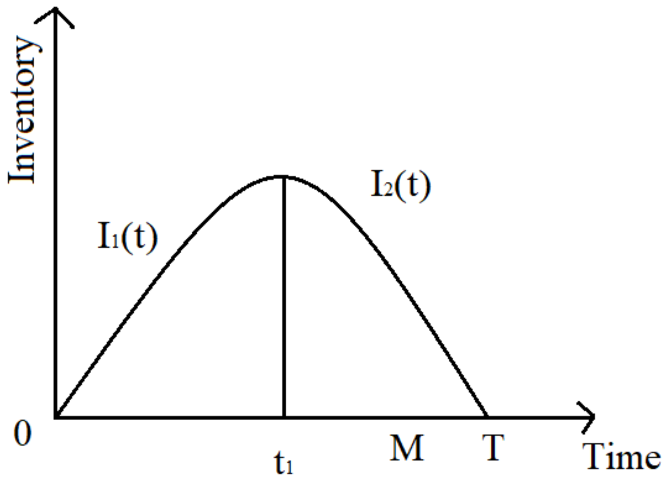

Fig. 1. Graphical representation of the proposed inventory level

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = R - D(p,\gamma), \quad 0 \le t \le t_1 \tag{1}$$

with $I_1(0) = 0$ and

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -D(p,\gamma), \quad t_1 \le t \le T \tag{2}$$

with $I_2(T) = 0$. Solving equations (1) & (2), we get

$$\begin{cases} I_1(t) = \left(a + b\alpha^\gamma p^{-\beta} - \alpha^\gamma p^{-\beta}\right)te^{-vt^2}, \\ I_2(t) = \alpha^\gamma p^{-\beta}(T-t)e^{-vt^2}. \end{cases} \tag{3}$$

Then, at $t = t_1$we have $I_1(t)$ is continuous, therefore, t_1 can be given by

$$t_1 = \frac{\alpha^\gamma p^{-\beta}T}{(a + b\alpha^\gamma p^{-\beta})}. \tag{4}$$

The maximum inventory level, denoted by Q, is given as:

$$Q = \left(a + b\alpha^\gamma p^{-\beta} - \alpha^\gamma p^{-\beta}\right)t_1 e^{-vt_1^2}. \tag{5}$$

Now, in this inventory model, several cost-related cases may be considered, as follows:

1. Setup cost is S.
2. Holding cost is

$$h\left[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt\right] = h\left[\left\{a + \alpha^\gamma p^{-\beta}(b-1)\right\}\frac{t_1^2}{2} + \alpha^\gamma p^{-\beta}\left(\frac{t_1^2}{2} - t_1T - \frac{T^2}{2}\right)\right]. \tag{6}$$

3. Deterioration cost is

$$c\left[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt\right] = c\left[\left\{a + \alpha^\gamma p^{-\beta}(b-1)\right\}\frac{t_1^2}{2} + \alpha^\gamma p^{-\beta}\left(\frac{t_1^2}{2} - t_1T - \frac{T^2}{2}\right)\right]. \tag{7}$$

4. Carbon emission cost related to production is
$$c_{ep}\int_0^{t_1} I_1(t)dt\ = c_{ep}\left[\left\{a+\alpha^\gamma p^{-\beta}(b-1)\right\}\frac{t_1^2}{2}\right]. \quad (8)$$
Carbon emission cost related to holding of inventory is
$$c_{eh}\left[\int_0^{t_1} I_1(t)dt+\int_{t_1}^{T} I_2(t)dt\right]=c_{eh}\left[\left[\left\{a+\alpha^\gamma p^{-\beta}(b-1)\right\}\frac{t_1^2}{2}+\alpha^\gamma p^{-\beta}\left(\frac{t_1^2}{2}-t_1T-\frac{T^2}{2}\right)\right]\right]. \quad (9)$$
Carbon emission cost related to transportation is
$$c_{et}\int_0^{t_1} I_1(t)dt=c_{et}\left[\left\{a+\alpha^\gamma p^{-\beta}(b-1)\right\}\frac{t_1^2}{2}\right]. \quad (10)$$
Hence, total carbon emission cost is
$$\left(c_{ep}+c_{eh}+c_{et}\right)\left[\left\{a+\alpha^\gamma p^{-\beta}(b-1)\right\}\frac{t_1^2}{2}\right]+c_{eh}\alpha^\gamma p^{-\beta}\left(\frac{t_1^2}{2}-t_1T-\frac{T^2}{2}\right).$$
Carbon emission reduction cost is
$$C_t e^{-zg}\times[Total\ carbon\ emission]$$
$$=C_t e^{-zg}\left[\left(c_{ep}+c_{eh}+c_{et}\right)\left[\left\{a+\alpha^\gamma p^{-\beta}(b-1)\right\}\frac{t_1^2}{2}\right]+c_{eh}\alpha^\gamma p^{-\beta}\left(\frac{t_1^2}{2}-t_1T-\frac{T^2}{2}\right)\right]. \quad (11)$$
5. Interest payable (IP): Two relations are considered based on the values of T and M: $T\geq M$ and $T\leq M$.

Case-I: When $M\leq T$: Interest payable during [M, T] is
$$cI_e\int_0^{T}(T-t)d(p)dt=cI_e\alpha^\gamma p^{-\beta}\frac{T^2}{2} \quad (12)$$
Interest charged: In this case, interest charged during 0 to M is
$$cI_c\int_M^{T} I_1(t)\,dt=cI_c\left\{a+\alpha^\gamma p^{-\beta}(b-1)\right\}\left\{\frac{T^2}{2}-\frac{M^2}{2}\right\} \quad (13)$$
Hence, the total average cost is given by
$$TC_1(T,t_1)=\frac{1}{T}[\text{Setup cost}+\text{Holding cost}+\text{Deterioration cost}$$
$$+\text{Carbon emission reduction cost}+\text{Interest charged}-\text{interest earned}]. \quad (14)$$
Therefore,
$$TC_1(T,t_1)=\frac{S}{T}+\frac{(h+c)}{T}\left[\left\{a+\alpha^\gamma p^{-\beta}(b-1)\right\}\frac{t_1^2}{2}+\alpha^\gamma p^{-\beta}\left(\frac{t_1^2}{2}-t_1T-\frac{T^2}{2}\right)\right]$$
$$+\frac{C_t e^{-zg}}{T}\left[\left(c_{ep}+c_{eh}+c_{et}\right)\left[\left\{a+\alpha^\gamma p^{-\beta}(b-1)\right\}\frac{t_1^2}{2}\right]+c_{eh}\alpha^\gamma p^{-\beta}\left(\left(\frac{t_1^2}{2}-t_1T-\frac{T^2}{2}\right)\right)\right]$$
$$+\frac{cI_c}{T}\left\{a+\alpha^\gamma p^{-\beta}(b-1)\right\}\left\{\frac{T^2}{2}-\frac{M^2}{2}\right\}-\frac{cI_e\alpha^\gamma p^{-\beta}\frac{T^2}{2}}{T}. \quad (15)$$

Case-II: When $M>T$: In this case, payable interest is zero.
Interest earned: Interest earned in this case
$$cI_e\left[\int_0^{T}(T-t)D(p,\gamma)dt-(M-T)\int_0^{T} D(p,\gamma)dt\right]=cI_e\left(\frac{3}{2}T^2-MT\right)\alpha^\gamma p^{-\beta}.$$
Hence, total average cost for this case is given by
$$TC_2(T,t_1)=\frac{1}{T}[\text{Setup cost}+\text{Holding cost}+\text{Deterioration cost}$$
$$+\text{Carbon emission reduction cost}+\text{Interest charged}-\text{interest earned}]. \quad (16)$$

Therefore, by inserting all the values, we get

$$TC_2(T,t_1)=\frac{S}{T}+\frac{(h+c)}{T}\left[\left\{a+\alpha^{\gamma}p^{-\beta}(b-1)\right\}\frac{t_1^2}{2}+\alpha^{\gamma}p^{-\beta}\left(\frac{t_1^2}{2}-t_1T-\frac{T^2}{2}\right)\right]$$
$$+\frac{C_te^{-zg}}{T}\left[\left(c_{ep}+c_{eh}+c_{et}\right)\left[\left\{a+\alpha^{\gamma}p^{-\beta}(b-1)\right\}\frac{t_1^2}{2}\right]+c_{eh}\alpha^{\gamma}p^{-\beta}\left(\frac{t_1^2}{2}-t_1T-\frac{T^2}{2}\right)\right]$$
$$-\frac{cI_e}{T}\left(\frac{3}{2}T^2-MT\right)\alpha^{\gamma}p^{-\beta}. \quad (17)$$

5. Solution procedure

We can observe that equations (15) and (17) are a non-linear function of T and t_1. Hence, the following procedure is applied for finding the optimal solutions:

Step (1): Enter value of each and every parameter in equations (15) and (17).

Step (2): In order to determine the stationary points, derivatives are evaluated for the objective functions, and each derivative is then set to zero, i.e.,

$$\begin{cases}\frac{\partial(TC_1)}{\partial t_1}=0\\ \frac{\partial(TC_1)}{\partial T}=0\end{cases}\text{and}\begin{cases}\frac{\partial(TC_2)}{\partial t_1}=0\\ \frac{\partial(TC_2)}{\partial T}=0\end{cases}. \quad (18)$$

By solving the aforementioned system of equations, we are able to determine the values T and t_1. Here,

$$\frac{\partial TC_1}{\partial t_1}=\frac{(h+c)t_1}{T}\left[a+\alpha^{\gamma}p^{-\beta}(b-1)\right]+\alpha^{\gamma}p^{-\beta}(t_1-T)\frac{(h+c)}{T}$$
$$+\frac{C_te^{-zg}}{T}\left(c_{ep}+c_{eh}+c_{et}\right)\left[\alpha^{\gamma}p^{-\beta}(b-1)t_1\right]+\frac{C_te^{-zg}}{T}\frac{c_{eh}\alpha^{\gamma}p^{-\beta}}{T}(t_1-T), \quad (19)$$

$$\frac{\partial TC_1}{\partial T}=-\frac{S}{T^2}-\frac{(h+c)}{T^2}\left[\left\{a+\alpha^{\gamma}p^{-\beta}(b-1)\right\}\frac{t_1^2}{2}+\alpha^{\gamma}p^{-\beta}\left(\frac{t_1^2}{2}-t_1T-\frac{T^2}{2}\right)\right]$$
$$+\frac{(h+c)}{T}\left[\alpha^{\gamma}p^{-\beta}(t_1-T)\right]-\frac{C_te^{-zg}}{T^2}\left[\left(c_{ep}+c_{eh}+c_{et}\right)\left\{a+\alpha^{\gamma}p^{-\beta}(b-1)\right\}\frac{t_1^2}{2}\right]$$
$$-\frac{C_te^{-zg}}{T}\frac{c_{eh}\alpha^{\gamma}p^{-\beta}}{T}(t_1+T)-\frac{2C_te^{-zg}c_{eh}\alpha^{\gamma}p^{-\beta}}{T^3}\left(\frac{t_1^2}{2}-t_1T-\frac{T^2}{2}\right)$$
$$-\frac{cI_c}{T^2}\left\{a+\alpha^{\gamma}p^{-\beta}(b-1)\right\}\left\{\frac{T^2}{2}-\frac{M^2}{2}\right\}+cI_c\left\{a+\alpha^{\gamma}p^{-\beta}(b-1)\right\}-\frac{cI_e\alpha^{\gamma}p^{-\beta}}{2}, \quad (20)$$

$$\frac{\partial TC_2}{\partial t_1}=\frac{(h+c)}{T}\left[\alpha^{\gamma}p^{-\beta}(b-1)t_1+\alpha^{\gamma}p^{-\beta}(t_1-T)\right]$$
$$+\frac{C_te^{-zg}}{T}\left[\left(c_{ep}+c_{eh}+c_{et}\right)\left\{\alpha^{\gamma}p^{-\beta}(b-1)t_1\right\}\right]+\frac{C_te^{-zg}}{T}\frac{c_{eh}\alpha^{\gamma}p^{-\beta}}{T}(t_1-T), \quad (21)$$

$$\frac{\partial TC_2}{\partial T}=-\frac{S}{T^2}-\frac{(h+c)}{T^2}\left[\left\{a+\alpha^{\gamma}p^{-\beta}(b-1)\right\}\frac{t_1^2}{2}+\alpha^{\gamma}p^{-\beta}\left(\frac{t_1^2}{2}-t_1T-\frac{T^2}{2}\right)\right]$$
$$+\frac{(h+c)}{T}\left[\alpha^{\gamma}p^{-\beta}(t_1-T)\right]-\frac{C_te^{-zg}}{T^2}\left[\left(c_{ep}+c_{eh}+c_{et}\right)\left\{a+\alpha^{\gamma}p^{-\beta}(b-1)\right\}\frac{t_1^2}{2}\right]$$
$$-\frac{C_te^{-zg}}{T}\frac{c_{eh}\alpha^{\gamma}p^{-\beta}}{T}(t_1+T)-\frac{2C_te^{-zg}c_{eh}\alpha^{\gamma}p^{-\beta}}{T^3}\left(\frac{t_1^2}{2}-t_1T-\frac{T^2}{2}\right)-\frac{3}{2}cI_e\alpha^{\gamma}p^{-\beta}. \quad (22)$$

Step (3): To determine whether the objective function is convex, let us formulate a Hessian matrix H_1 and H_2, as followes:

$$H_1 = \begin{bmatrix} \frac{\partial^2(TC_1)}{\partial(t_1)^2} & \frac{\partial^2(TC_1)}{\partial t_1 \partial T} \\ \frac{\partial^2(TC_1)}{\partial T \partial t_1} & \frac{\partial^2(TC_1)}{\partial(T)^2} \end{bmatrix} \text{ and } H_2 = \begin{bmatrix} \frac{\partial^2(TC_2)}{\partial(t_1)^2} & \frac{\partial^2(TC_2)}{\partial t_1 \partial T} \\ \frac{\partial^2(TC_2)}{\partial T \partial t_1} & \frac{\partial^2(TC_2)}{\partial(T)^2} \end{bmatrix} \tag{23}$$

The principal minor values are calculated from the Hessian matrix in the manner described below:

$M_{11} = \left[\frac{\partial^2 TC_1}{\partial {t_1}^2}\right]_{(T,t_1)}$, $M_{22} = \begin{bmatrix} \frac{\partial^2 TC_1}{\partial {t_1}^2} & \frac{\partial^2 TC_1}{\partial t_1 \partial T} \\ \frac{\partial^2 TC_1}{\partial T \partial t_1} & \frac{\partial^2 TC_1}{\partial T^2} \end{bmatrix}_{(T,t_1)}$ for H_1 and

$M_{11} = \left[\frac{\partial^2 TC_2}{\partial {t_1}^2}\right]_{(T,t_1)}$, $M_{22} = \begin{bmatrix} \frac{\partial^2 TC_2}{\partial {t_1}^2} & \frac{\partial^2 TC_2}{\partial t_1 \partial T} \\ \frac{\partial^2 TC_2}{\partial T \partial t_1} & \frac{\partial^2 TC_2}{\partial T^2} \end{bmatrix}_{(T,t_1)}$ for H_2, respectively.

The derivatives are given as

$$\frac{\partial^2 TC_1}{\partial^2 t_1} = \frac{(h+c)}{T}\left[a + \alpha^\gamma p^{-\beta}(b-1)\right] + \alpha^\gamma p^{-\beta}\frac{(h+c)}{T} + \frac{C_t e^{-zg}}{T}\left(c_{ep} + c_{eh} + c_{et}\right)\left[\alpha^\gamma p^{-\beta}(b-1)\right] + \frac{C_t e^{-zg}}{T}\frac{c_{eh}\alpha^\gamma p^{-\beta}}{T}, \tag{24}$$

$$\frac{\partial^2 TC_1}{\partial T \partial t_1} = -\frac{(h+c)t_1}{T^2}\left[a + \alpha^\gamma p^{-\beta}(b-1)\right] - \frac{(h+c)}{T^2}\alpha^\gamma p^{-\beta}(t_1 - T) - \alpha^\gamma p^{-\beta}\frac{(h+c)}{T} - \frac{C_t e^{-zg}}{T^2}\left(c_{ep} + c_{eh} + c_{et}\right)\left[\alpha^\gamma p^{-\beta}(b-1)t_1\right] - \frac{2C_t e^{-zg} c_{eh}\alpha^\gamma p^{-\beta}}{T^3}(t_1 - T) - \frac{C_t e^{-zg}}{T}\frac{c_{eh}\alpha^\gamma p^{-\beta}}{T}, \tag{25}$$

$$\frac{\partial^2 TC_1}{\partial t_1 \partial T} = -\frac{(h+c)}{T^2}\left[\left\{a + \alpha^\gamma p^{-\beta}(b-1)t_1\right\} + \alpha^\gamma p^{-\beta}(t_1 - T)\right] + \frac{(h+c)}{T}\alpha^\gamma p^{-\beta} - \frac{C_t e^{-zg}}{T^2}\left[\left(c_{ep} + c_{eh} + c_{et}\right)\left\{a + \alpha^\gamma p^{-\beta}(b-1)\right\}t_1\right] - \frac{C_t e^{-zg}}{T}\frac{c_{eh}\alpha^\gamma p^{-\beta}}{T} - \frac{2C_t e^{-zg} c_{eh}\alpha^\gamma p^{-\beta}}{T^3}(t_1 - T), \tag{26}$$

$$\frac{\partial^2 TC_1}{\partial^2 T} = \frac{2s}{T^3} + \frac{2(h+c)}{T^3}\left[\left\{a + \alpha^\gamma p^{-\beta}(b-1)\right\}\frac{t_1^2}{2} + \alpha^\gamma p^{-\beta}\left(\frac{t_1^2}{2} - t_1 T - \frac{T^2}{2}\right)\right] + \frac{(h+c)}{T^2}\alpha^\gamma p^{-\beta}(t_1 + T) - \frac{(h+c)}{T^2}\alpha^\gamma p^{-\beta}(t_1 - T) + \frac{(h+c)}{T}\alpha^\gamma p^{-\beta} + \frac{6C_t e^{-zg} c_{eh}\alpha^\gamma p^{-\beta}}{T^4}\left(\frac{t_1^2}{2} - t_1 T - \frac{T^2}{2}\right) + \frac{2C_t e^{-zg} c_{eh}\alpha^\gamma p^{-\beta}}{T^3}(t_1 + T) - \frac{C_t e^{-zg}}{T}\frac{c_{eh}\alpha^\gamma p^{-\beta}}{T} + \frac{2cI_c}{T^3}\left\{a + \alpha^\gamma p^{-\beta}(b-1)\right\}\left\{\frac{T^2}{2} - \frac{M^2}{2}\right\} - \frac{cI_c}{T}\left\{a + \alpha^\gamma p^{-\beta}(b-1)\right\}, \tag{27}$$

and

$$\frac{\partial^2 TC_2}{\partial^2 t_1} = \frac{(h+c)}{T}\left[\alpha^\gamma p^{-\beta}(b-1) + \alpha^\gamma p^{-\beta}\right] + \frac{C_t e^{-zg}}{T}\left[\left(c_{ep} + c_{eh} + c_{et}\right)\left\{\alpha^\gamma p^{-\beta}(b-1)\right\}\right] + \frac{C_t e^{-zg}}{T}\frac{c_{eh}\alpha^\gamma p^{-\beta}}{T}. \tag{28}$$

Again,

$$\frac{\partial^2 TC_2}{\partial T \partial t_1} = -\frac{(h+c)}{T^2}\left[\alpha^\gamma p^{-\beta}(b-1)t_1 + \alpha^\gamma p^{-\beta}(t_1 - T)\right] - \frac{(h+c)}{T}\alpha^\gamma p^{-\beta}$$
$$-\frac{C_t e^{-zg}}{T^2}\left[\left(c_{ep}+c_{eh}+c_{et}\right)\{\alpha^\gamma p^{-\beta}(b-1)t_1\}\right] - \frac{2C_t e^{-zg} c_{eh}\alpha^\gamma p^{-\beta}}{T^3}(t_1 - T) - \frac{C_t e^{-zg}}{T}\frac{c_{eh}\alpha^\gamma p^{-\beta}}{T}, \quad (29)$$

$$\frac{\partial^2 TC_2}{\partial t_1 \partial T} = -\frac{(h+c)}{T^2}\left[\{a + \alpha^\gamma p^{-\beta}(b-1)\}t_1 + \alpha^\gamma p^{-\beta}(t_1 - T) + \frac{(h+c)}{T}\alpha^\gamma p^{-\beta}\right]$$
$$-\frac{C_t e^{-zg}}{T^2}\left[\left(c_{ep}+c_{eh}+c_{et}\right)\{a + \alpha^\gamma p^{-\beta}(b-1)\}t_1\right] - \frac{C_t e^{-zg}}{T}\frac{c_{eh}\alpha^\gamma p^{-\beta}}{T} - \frac{2C_t e^{-zg} c_{eh}\alpha^\gamma p^{-\beta}}{T^3}(t_1 - T), \quad (30)$$

$$\frac{\partial^2 TC_2}{\partial^2 T} = \frac{2S}{T^3} + \frac{2(h+c)}{T^3}\left[\{a + \alpha^\gamma p^{-\beta}(b-1)\}\frac{t_1^2}{2} + \alpha^\gamma p^{-\beta}\left(\frac{t_1^2}{2} - t_1 T - \frac{T^2}{2}\right)\right]$$
$$+\frac{(h+c)}{T^2}\left[\alpha^\gamma p^{-\beta}(t_1 + T)\right] - \frac{(h+c)}{T^2}\left[\alpha^\gamma p^{-\beta}(t_1 - T)\right] - \frac{(h+c)}{T}\left[\alpha^\gamma p^{-\beta}\right]$$
$$+\frac{2C_t e^{-zg}}{T^3}\left[\left(c_{ep}+c_{eh}+c_{et}\right)\{a + \alpha^\gamma p^{-\beta}(b-1)\}\frac{t_1^2}{2}\right] + \frac{4C_t e^{-zg} c_{eh}\alpha^\gamma p^{-\beta}}{T^3}(t_1 + T)$$
$$-\frac{C_t e^{-zg}}{T}\frac{c_{eh}\alpha^\gamma p^{-\beta}}{T} + \frac{6C_t e^{-zg} c_{eh}\alpha^\gamma p^{-\beta}}{T^4}\left(\frac{t_1^2}{2} - t_1 T - \frac{T^2}{2}\right). \quad (31)$$

The objective function is minimum at points (T, t_1) and the positive definiteness is shown by the Hessian matrix.

Step (4): Therefore, extreme values of the objective functions are $TC_1(T^*, t_1^*)$, $TC_2(T^*, t_1^*)$ and (T^*, t_1^*) is a point of extreme values.

6. Numerical Example

Considering the following parametric values as initial conditions in Table 2.

Table 2
Parametric values are considered for numerical evaluation

Parameter	Value	Parameter	Value
s	200$	G	0.5$
h	0.3$	c_{ep}	3$
a	7	c_{eh}	2$
α	0.002	c_{et}	2$
p	10$	c	5$
β	0.2	I_c	0.15 $/unit time
b	6	I_e	0.13 $/unit time
c_t	1	M	0.2 unit time
z	0.01		

We get the following result, shown in Table 3, as follows:

Table 3
Evaluated results by numerical evaluation

Cases	t_1	T	Z
Case-I	0.08447	1.50402	265.756
Case-II	0.08649	1.53993	259.830

The graphical views of the cost functions are represented in Figures 2 and 3, respectively. The convexities of the cost functions in both cases are shown more clearly in the figures.

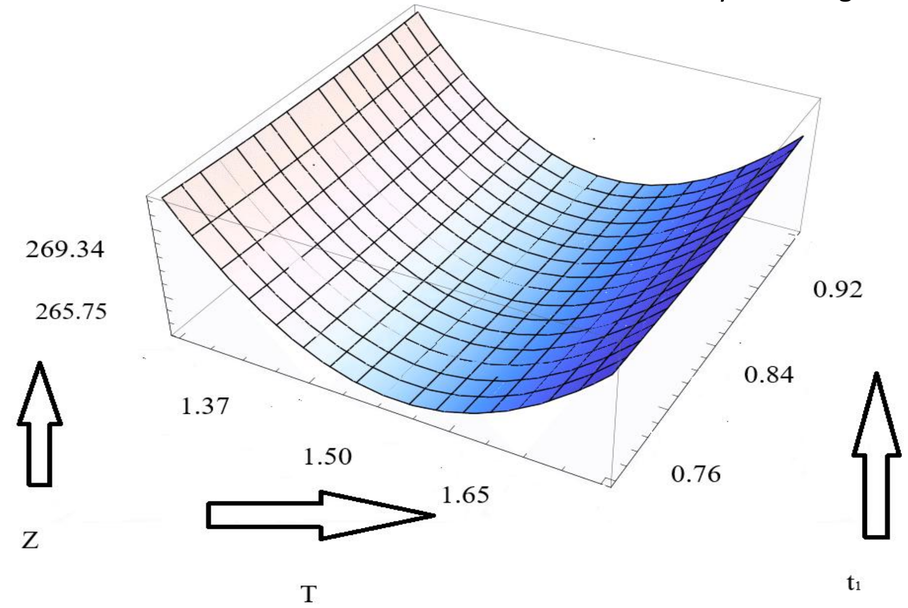

Fig. 2. Graphical diagram of the cost function with its convexity for Case I

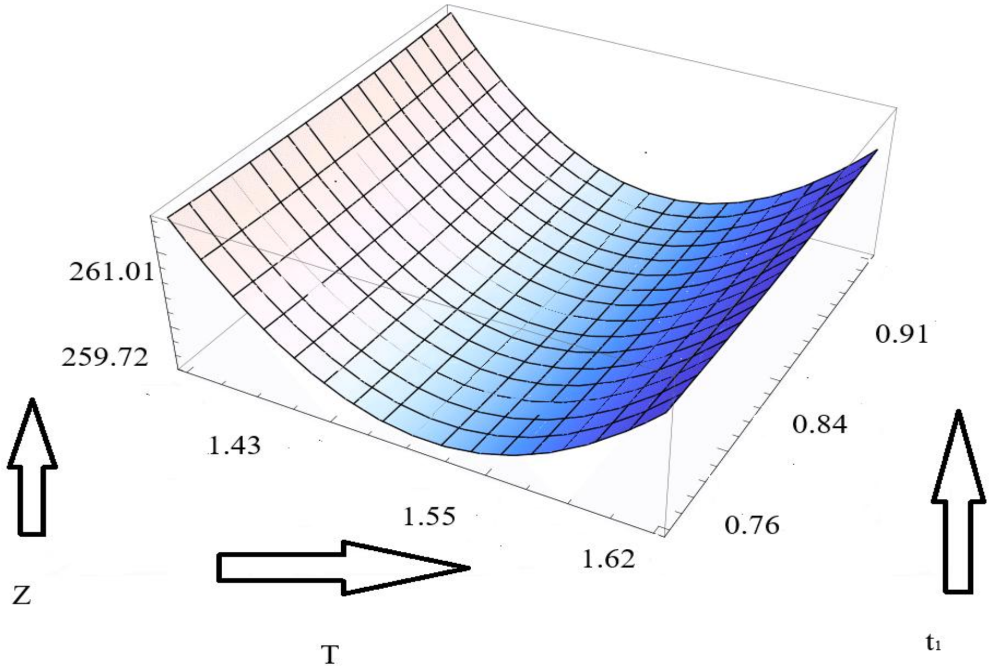

Fig. 3. Graphical diagram of the cost function with its convexity for Case II

6.1 Sensitivity Analysis

This section presents the sensitivity analysis in detail and its significance. The parametric values are changed by -20% to $+20\%$ with 10% difference to catch its changes. The impact has been observed in Table 4 with the parameters t_1, T and Z, respectively.

Table 4
Changes in the different parameter values and their impact on the results

Parameter	% change	t_1	T	Z
h	+20%	0.08448	1.50418	265.728
	+10%	0.08447	1.50410	265.742
	-10%	0.08447	1.50394	265.770
	-20%	0.08446	1.50386	265.784
α	+20%	0.08456	1.50388	265.780
	+10%	0.08452	1.50394	265.769
	-10%	0.08441	1.50409	265.742
	-20%	0.08435	1.50418	265.727
γ	+20%	0.08383	1.50493	265.594
	+10%	0.08480	1.50448	265.675
	-10%	0.08479	1.50355	265.838
	-20%	0.08511	1.50309	265.920
β	+20%	0.08302	1.57284	254.132
	+10%	0.08376	1.53806	259.876
	-10%	0.08514	1.47068	271.777
	-20%	0.08577	1.43805	277.941
S	+20%	0.09254	1.64767	291.139
	+10%	0.08859	1.57748	278.737
	-10%	0.08013	1.42677	252.108
	-20%	0.07554	1.34511	237.677
z	+20%	0.08451	1.50475	265.626
	+10%	0.08449	1.50438	265.691
	-10%	0.08445	1.50365	265.821
	-20%	0.08443	1.50328	265.880
p	+20%	0.08391	1.53092	261.088
	+10%	0.08418	1.51802	263.305
	-10%	0.08478	1.48867	268.494
	-20%	0.08512	1.47170	271.589
a	+20%	0.06864	1.38434	288.699
	+10%	0.07587	1.44046	277.466
	-10%	0.09484	1.57680	253.502
	-20%	0.10757	1.66130	240.621
b	+20%	0.07106	1.35056	295.936
	+10%	0.07721	1.42111	281.252
	-10%	0.09319	1.60335	249.297
	-20%	0.10390	1.72539	231.671

The following are the observations derived from Table 4, as follows:

i. If the parameter h increases, value of the parameters t_1 decrease, T increase and Z increases and decreases conversely.
ii. When the parameter α increases, then the parametric values of t_1 and T decrease and Z increases and vice versa.
iii. If γ increases, t_1, T and Z deceases. The values increase oppositely.
iv. When the parameter β increases, then t_1, T increases and Z decreases and vice versa.
v. If S increases, then the parameters t_1, T and Z increase. The values decrease conversely.
vi. When the parameter z increases, then t_1, T increases and Z decreases. The values increase oppositely.
vii. If p increases, then, the parameters t_1, T and Z increases and vice versa.
viii. When a increases, then, t_1, T decrease and Z increase. The values decrease conversely.

ix. If b increases, then, t_1, T decreases and Z increase and vice versa.

7. Managerial Insight

The mathematical model demonstrates that the firms can improve profitability when simultaneously reducing environmental impacts by integrating the reduction of carbon emissions strategies into inventory decisions. Managers should view sustainability as an opportunity to optimize long-term performance instead of a cost burden. Given that demand is dependent on both stock and price availability, managers must strategically set selling prices and stock amounts to influence customer perceptions and increase demand. Sales can be reduced due to overpricing or understocking, while underpricing or overstocking can increase holding and deterioration costs, respectively. The total inventory cost and replenishment strategy significantly affect the deterioration rate. Managers handling perishable or degradable goods must adopt shorter replenishment cycles and effective preservation techniques to minimize losses due to product spoilage.

Utilizing a trade credit period provides financial flexibility to both buyers and sellers. For the buyer, it reduces immediate cash flow pressure and allows reinvestment of available funds. For the seller, an appropriate credit policy can increase market competitiveness and attract more customers, improving demand. Incorporation of green technology can reduce carbon-related costs and improve brand image. Managers should consider environmental policies as integral to inventory and production decisions rather than as external constraints. Sensitivity analysis reveals that changes in parameters such as the deterioration rate, carbon emission cost and credit period can significantly affect the optimal profit. Managers should regularly reassess these parameters to maintain system efficiency under dynamic market and policy conditions. The model emphasizes that pricing, inventory control, payment policy, and carbon management are interdependent decisions. Managers should adopt an integrated optimization approach rather than handling these aspects separately to improve operational and environmental performance. These recommendations can be implemented by decision-makers to ensure a profitable and sustainable system.

8. Conclusions

This inventory model plays an important role in aligning demand and supply with market trends. The production rate of products has been controlled by demand, and the amount of deterioration of their products has been controlled. Here, a mathematical model and sensitivity analysis for critical evaluation are well presented. This model has been applied appropriately by collecting the original research data through realistic market visits. Price elasticity is an important variable in how sales revenue and profitability flow. Stock elasticity controls and determines how companies in the market adapt their supply, branding, and expansion strategies to consumer demand as market conditions change. This study aims to reduce carbon emissions effectively to mitigate negative environmental impacts. In addition, the product has the advantage of repaying the principal on time, which is an important quality criterion.

The results reveal that both pricing and stocking strategies significantly influence demand and overall profitability. The inclusion of a permissible payment delay enhances the buyer's financial flexibility and supports better cash flow management, while increasing the seller's market competitiveness. Moreover, carbon-emission reduction efforts, although requiring an initial investment, contribute to long-term sustainability and cost savings by minimizing environmental penalties and enhancing corporate reputation.

Sensitivity analysis indicates that parameters such as deterioration rate, credit period, and emission cost significantly affect optimal solutions, underscoring the importance of continuously monitoring these variables in practice. The model thus provides valuable insights for managers seeking to design sustainable, profitable inventory systems under market-sensitive and environmentally constrained conditions.

Future research could show how demand can be driven by digital technologies and dynamic markets. Additionally, machine learning (ML) and artificial intelligence (AI)- based forecasts can be used to improve demand forecasting by incorporating customer behaviour, competitive pricing, and rail-time data. All of these future inventory models are combined with multi-echelon supply chains, in which item stock visibility more directly affects demand.

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Conflicts of Interest

The authors declare no conflicts of interest.

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